Spin Rotator Design for SuperKEKB

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(Dated: August 9, 2019)

Spin-polarized accelerators are interesting for many physics purposes, however, there aren’t many polarized machines in existence. Spin rotators are a crucial component to maintain a spin-polarized beam. This study aims to design a spin rotator for SuperKEKB electron beam using dipole-solenoid combine function magnets and skew quadrupoles. Using Bmad and Tao, we modeled and optimized the spin rotator and kept it transparent to the lattice.

1. INTRODUCTION

1.1 Spin

Spin is the intrinsic angular momentum of particles and is purely quantum mechanical. It plays a critical role in many of the particle physics processes. Having a spin-polarized beam in collision experiments serves many physics purposes. Polarization can increase the cross-section of some rare event and thus the frequency; other studies concern the polarization dependence or asymmetry between identical processes. This results in an increasing interest in the spin-polarized beam at high-energy and nuclear physics facilities.

1.2 SuperKEKB

SuperKEKB is a 7 GeV electron - 4 GeV positron collider located in Tsukuba, Ibaraki Prefecture, Japan. It is an upgrade to the KEKB accelerator, achieving a luminosity 40 times higher than that of KEKB, to seek new physics beyond the Standard Model. SuperKEKB consists of two storage rings: the High Energy Ring (HER) for electron beam and the Low Energy Ring (LER) for positron beam. In this project, we aim to design a spin rotator for the High Energy Ring of SuperKEKB as part of the effort to polarize the electron beam.

2. METHOD

2.1 Spin Motion

Despite the quantum mechanical nature of spin, the spin motion of beams in accelerators can be analyzed largely based on classical equations of motion, similar to the rigid body motion. The kinematics of classical spin motion in an electromagnetic field is governed by the Thomas-BMT equation

\[
\frac{d\vec{P}}{dt} = \vec{\Omega} \times \vec{P} \tag{1}
\]
where the axial vector $\vec{\Omega}$ is given by

$$\vec{\Omega} = -\frac{e}{m\gamma} \left[ (1 + G\gamma)\vec{B}_\perp + (1 + G)\vec{B}_\parallel + (G\gamma + \frac{\gamma}{\gamma + 1})\frac{\vec{E} \times \vec{v}}{c^2} \right]. \quad (2)$$

In the above equation, $e, m$ are the charge and mass of the particle, $\gamma$ is the Lorentz factor, $\vec{v}$ the velocity, $c$ the speed of light, $\vec{E}$ and $\vec{B}$ the electric and magnetic fields, and $G = (g - 2)/2$ is the gyromagnetic anomaly. The vector $\vec{P}$ can be considered as the polarization of an ensemble of particles or as a classical representation of the spin of a single particle [2].

It can be shown that for a particle on a closed orbit in a circular machine there exists a closed solution for the spin motion as well. This defines an axis, commonly called $\hat{n}_0$ that is called the stable spin direction. A polarization vector parallel to $\hat{n}_0$ remains the same after turns. $\hat{n}_0$ is usually close to vertical due to the vertical guide field [3]. The polarization needs to be vertical in the ring to be close to $\hat{n}_0$ and longitudinal at the interaction point, which will be achieved by installing the spin rotators on both sides of the interaction point.

### 2.2 Spin Rotator Model

The spin rotator magnets are modeled as a series of 6 solenoid-dipole combined function magnets that control the spin motion. A quadrupole at an arbitrary roll angle is added on top of each section to compensate for the plane coupling introduced by the solenoid field [4]. The whole spin rotator consists of three such magnets, two identical and one different, that together rotate the spin to the horizontal plane. The lattice dipoles between the spin rotator and the interaction point then rotate the spin to the longitudinal direction. These three spin rotator magnets will replace three existing dipoles with the same bending angles to preserve the geometry of the machine.
2.3 Tools

To model the spin rotator, we used Bmad and Tao developed at Cornell University by David Sagan et al. Bmad is a subroutine library for relativistic charged-particle and X-Ray simulations in accelerators and storage rings. Tao is a general-purpose program for simulating high energy particle beams using Bmad library [5]. We used distribution 2019.0523 for all results in this article and the spin rotator magnets are modeled by element Sol_quad with attribute hkick.

3. DETAILED RESULTS

3.1 Original Lattice

Assuming the spin points in the longitudinal direction, the existing dipoles will rotate its orientation in the horizontal plane by $\pi/2$ inside the second BLA4LE dipole. At this point it requires a rotation of $\pi/2$ around the longitudinal axis. Hence, we choose to replace two BLA4LE and one B2E in this region with our spin rotator magnets, as shown in figure 2. We will call the spin rotator magnets BLA4LE and B2E, the dipole they are replacing, in the following discussion.
3.2 Stand-alone Model

We first made stand-alone models for BLA4LE and B2E with approximated solenoid strength fixed at \( ks = 0.095734, 0.02077 \text{ m}^{-1} \). We allowed the 6 quadrupole strengths and roll angles to change and minimized the plane coupling by reducing the 2-by-2 \( C \) matrix. The Sol_quad element in Bmad is considered as a straight element, therefore the dipole field \((hkick)\) results in an artificial orbit excursion. The reference frame is rotated after each section using patch elements for the beam to enter through the center of the next section. This sawtooth-shaped orbit excursion causes the beam to experience the quadrupole and solenoid field in the simulation, leading to small errors which will be reduced later in the slice model.

We used lmdif to optimize the quadrupole parameters. The fitting is very sensitive to the initial conditions and cannot guarantee to find the global minimum, therefore it is necessary to start at different conditions and compare the results. We tried setting the initial strengths to \( \pm 0.1 \) and the initial angle to 0, \( \pi/2 \). Some initial points lead to decoupled solutions while others don’t. Additionally, a limit on the maximum of quadrupole strength is enforced to reduce unnecessary focusing power. We empirically found that the maximum strength needs to be at least 0.5 to have a decoupled solution.

In the end, we were able to fully decouple the planes with quadrupoles of various strengths up to \( k1 = 0.5 \text{ m}^{-2} \) or 11.67 T/m. This demonstrates the possibility of decoupling a spin rotator using skew quadrupoles within reasonable field strength.
3.3 Solenoid Strength Fit

The stand-alone models are plugged in a partial lattice from the interaction point to the end of B2E. During this step, we varied the two solenoid strength (two BLA4LE are kept identical) to minimize the x and z component of spin after the rotator. Quadruples are turned off to avoid the effect of the quadrupole field felt by the excursions. The maximum solenoid strength is also enforced to a minimum during the optimization. We found that the solenoid strength for BLA4LE and B2E are respectively $k_s = 0.19363, 0.03417$ m$^{-1}$ or $B_s = 4.5, 0.798$ T. The quadrupole strength and angles are refitted correspondingly, with the extra constraint on the vertical dispersion to avoid increasing vertical emittance. Since the solenoid strength nearly doubled, stronger quadrupoles are required to obtain a good compensation. For B2E, all quadrupoles remained under $k_1 = 0.5$ m$^{-2}$. But for BLA4LE, the new minimized maximum quadrupole strength is $k_1 = 1.5$ m$^{-2}$ or 35 T/m, which requires a 2.8 T magnetic field at r = 8 cm. At the end of each spin rotator magnet, the x-y planes are fully decoupled and the vertical dispersion is minimized.

3.4 Match to Lattice

We inserted the two stand-alone models into the existing lattice, sher.5780, and control the quadrupoles near the spin rotator magnets to match Twiss parameters and horizontal dispersion such that the spin rotator is transparent to the system. The beta functions and dispersions must also be limited to a sensible range. Quadrupole QLA4LE to QD5E were altered and noted with additional A/B in the name. We first used QLA6LE to QLA10LE to bring the beta functions between 150-170 m to a reasonable range (<1000 m), then used QLA9LE to QD3E to further control the beta functions between 170-190 m to lower values (<400 m). Using this result as a starting point, we successfully fit the strengths of the triplet and quadruplet around the B2E dipole to match the beta, alpha and horizontal dispersion to the values in the original lattice. The overall R matrix shows that the off-diagonal coupling terms are reasonably small.

\[
\begin{bmatrix}
11.33 & -0.3086 & -3.098 \times 10^{-5} & -4.25 \times 10^{-6} & 0 & 0.2377 \\
1.096 & 0.05839 & -2.69 \times 10^{-6} & -4.4 \times 10^{-7} & 0 & -0.0410 \\
-6.65 \times 10^{-6} & 7.01 \times 10^{-6} & -18.82 & -0.08484 & 0 & -0.06507 \\
4.50 \times 10^{-6} & -5.07 \times 10^{-6} & 13.24 & 6.549 \times 10^{-3} & 0 & 0.00378 \\
0.7377 & 8.929 \times 10^{-4} & -0.7904 & -1.057 \times 10^{-4} & 1 & -0.05413 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \]

Table. 1. R matrix of the spin rotator.

3.5 Slice Model

To decrease the effect of orbit excursion, we sliced each section of the magnet into 4 pieces and fix the orbit after each slice. This effectively reduces the excursion by a factor of 16. Section 3.2-3.4 were repeated for the slice model. The quadrupole strengths are within the same upper limit. The slice model provides several additional benefits to the design. The beta functions are more well-behaved than the unsliced model because the erroneous influence of strong quadrupoles is much smaller. The horizontal dispersion is also greatly reduced in the new model. However, the vertical dispersion at the end of the rotator becomes significant and cannot be corrected in the lattice. This results in a large oscillation of vertical dispersion in the closed orbit solution, which jeopardizes the vertical emittance. To address this issue, we relaxed the limit of quadrupole strength in B2E from
$k_1 = 0.5$ to $1.2 \text{ m}^{-2}$ and used the degrees of freedom to diminish the vertical dispersion. The final result successfully restores the vertical dispersion to nearly 0, and no significant vertical dispersion appears throughout the lattice.

4. CONCLUSION

4.1 Comparison of Parameters

The comparison of the lattice with and without the spin rotator is shown in figure 4. Since the Twiss parameters and dispersions are matched, the spin rotator is transparent to the rest of the lattice. The modeled spin rotator affects the machine parameters such as emittance, damping and momentum compaction. A comparison is given in the following table. The vertical emittance is around 9 pm, a factor of 5 increase from the original. The chromaticity becomes dramatic with the spin rotator due to strong quadrupoles, which needs to be addressed in future work.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without Rotator</th>
<th>With Rotator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emittance x</td>
<td>4.44×10^{-9}</td>
<td>5.42×10^{-9}</td>
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<tr>
<td>Emittance y</td>
<td>1.88×10^{-12}</td>
<td>9.70×10^{-12}</td>
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<tr>
<td>Alpha Damp x</td>
<td>1.79×10^{-4}</td>
<td>2.54×10^{-4}</td>
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<tr>
<td>Alpha Damp y</td>
<td>1.79×10^{-4}</td>
<td>1.81×10^{-4}</td>
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<tr>
<td>Damping Partition x</td>
<td>0.999667</td>
<td>1.420164</td>
</tr>
<tr>
<td>Damping Partition y</td>
<td>1.001851</td>
<td>1.013181</td>
</tr>
<tr>
<td>Chromaticity x</td>
<td>3.953515</td>
<td>-82.933671</td>
</tr>
<tr>
<td>Chromaticity y</td>
<td>6.595477</td>
<td>-17.182644</td>
</tr>
<tr>
<td>Momentum Compaction</td>
<td>4.54×10^{-4}</td>
<td>4.58×10^{-4}</td>
</tr>
</tbody>
</table>

Table. 2. Comparison of machine parameters.

4.2 Future Work

This project demonstrates the possibility of building a compact spin rotator using dipole and solenoid by skew quadrupoles. The model could be applied to upgrade SuperKEKB without changing the geometry of the ring or provide an alternative design for future polarized machines such as Electron-Ion Collider (EIC). Since the SuperKEKB lattice is not symmetric, we would need to fit the other spin rotator and calculate the spin tune. The issue with strong focusing power and chromaticity also needs to be addressed.

ACKNOWLEDGEMENT

I would like to thank my mentor Uli Wienands for his help and guidance throughout the project; David Sagan, Cornell U. for help with Bmad; Lee Teng internship for the opportunity to participate in the research; Kathy Harkay and Peter Garbincius for organizing the internship. This work is supported by U. S. Department of Energy, Office of Science, under Contract No. DE-AC02-06CH11357.

REFERENCES