Material properties data for heat transfer modeling in Nb$_3$Sn magnets

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Abstract

A Network Model is used to study the thermal behavior of magnet coils and to calculate the quench levels of the superconducting magnets. The cryogenic materials properties data are essential input for heat transfer calculation in superconducting magnets. In order to prepare this input and to study of model sensitivity on different material properties parameterizations a material data compilation is required. The collected data will be implemented into thermal magnet model to calculate the thermal behavior of superconducting magnets as well as to assess the magnet components crucial to heat transfer in the magnet.

1. Introduction

The accelerator superconducting magnets quench, especially in Large Hadron Collider (LHC) [1], is undesirable. In order to minimize the number of quenches, one needs to calculate the quench limit of each proposed magnet design.

This paper is focusing on material properties data required for thermal modeling of superconducting magnets, particularly on available material data at low temperatures as well as heat transfer in solid materials. The measured material properties in the
temperature range 1-300 K were collected and organized. The compiled dataset will be used inter alia as input to the thermal model of Nb₃Sn superconducting magnets.

### 1.1 Heat conductivity

Thermal conductivity is the property of a material's ability to conduct heat. Thermal conductivity in a steady state, unidirectional heat flow through an isotropic medium can be defined by the Fourier-Biot equation

$$\frac{Q}{A} = -k \frac{dT}{dx}$$  \hspace{1cm} \text{Eq. 1}

Where $Q$ [J] is rate of heat flow through area $A$ [m$^2$] with a temperature gradient, $dT/dx$ [K/m] and $k$ [W/mK] is thermal conductivity.

In order to get the total heat conductivity additional terms with linear temperature dependence can be added to Eq. 1. In metals and dilute alloys heat conductivity is dominated by free electrons. More highly alloyed metals have decreased free electron heat conductivity and in this case the phonons heat conduction dominates. In semiconducting materials electron-hole pair conduction contributes to the electron and phonon components. Phonons conduction is the primary heat transfer mechanism in all other non-metallic materials. Thermal conductivity is temperature dependent, in general decreases as the temperature is lowered.

At low temperatures the generic name, e.g. copper, titanium, carbon, of a material is generally not sufficient to characterize the thermal conductivity. This is because sample variables such as lattice imperfections, impurities, magnetic fields, size of sample, and shape of sample can all effect thermal conductivity by orders of magnitude. It is vital to pick the “best value” of a material of for a specific application. A good overview of thermal conductivity is given by Reed and Clark in [44].

### 1.2 Heat capacity

Heat capacity is defined as the amount of heat required to raise the temperature of a system by a unit of temperature. The SI unit of heat capacity is the J/K. It is defined as:

$$C = -\left(\frac{dq}{dT}\right)_x$$  \hspace{1cm} \text{Eq. 2}

where $q$ [J] is heat, $T$ [K] is temperature, and $x$ can be either volume or pressure.

Heat capacity is an extensive property, particular to the samples. It is useful to present heat capacity as an intensive variable, making it intrinsic characteristic of a particular substance, rather than sample dependent. In this case it is known as specific heat capacity. Specific heat is the heat capacity per unit mass of a material, [J/kg K]. Also molar heat capacity [J/mol K] or volumetric-specific heat capacity heat, [J/m$^3$ K]
are commonly used. In a gas molar heat capacity is more useful when in a solid either specific or volumetric-heat capacity is used, depending on the known information.

Specific heat is a strong function of temperature as can be seen in the experimental graphs in the Appendix. The experimental data show that below 100 K specific heat approximately decreases as $T^{-3}$ in most materials. At very low temperatures, below 10 K specific heat becomes linear and according to the third law of thermodynamics it goes to zero at 0 K. For temperatures above 100 K specific heat typically approaches a constant value [44].

The value of specific heats of most materials at room temperature is of order of 25 [J/mol K]. This number is known as the Dulong-Petit value [44]. This value holds well for elements that lack strong inter-atomic forces. Boltzmann justified this theoretically with the equipartition-of-energy theorem [44]. Equipartition theory states that in thermal equilibrium energy is shared equally among the degrees of freedom. In a solid, there are six degrees of freedom associated with lattice vibrations. Quantum effects become significant at low temperatures, resulting in equipartition-of-energy theorem to overestimate specific heat. This is because the difference between quantum energy levels in a degree of freedom exceeds the average thermal energy of the system. In this case, the degree of freedom is said to be frozen out. The result is that the degree of freedom cannot store thermal energy and can no longer contribute to the specific heat.

Einstein was the first to apply quantum concepts to the thermal vibrations of atoms and molecules in a solid crystal lattice. The quantized thermal vibrations have become known as phonons. He assumed that particles of a crystal lattice oscillated independently of one another at a single frequency. This theory works well for high temperatures but fails in the low temperature range. The problem with Einstein’s theory was the assumption of a single allowable frequency of vibration [44].

Debye treated a crystalline solid as an infinite elastic continuum and visualized the excitement of all the possible elastic standing waves in the material [44]. Debye also established a lower limit to the wavelengths that could exist in the crystal. Phonons are the chief means of storing energy in solids and thus dominate the heat capacity of most materials in cryogenic temperature range. The phonon contribution to the specific heat at constant volume $C_V$ may be estimated by the Debye function.

$$C_V = 9rR \left( \frac{T}{\theta_D} \right)^3 \int_0^{X_{\text{max}}} e^{X} \cdot \frac{X^4}{(e^X - 1)^2} dX$$  \hspace{1cm} \text{Eq. 3}

At high temperatures ($\theta_D/T<0.1$) this may be approximated by

$$C_V = 3rR \left( 1 - \frac{X^2}{20} + \frac{X^4}{560} - \frac{X^6}{18144} \right)$$  \hspace{1cm} \text{Eq. 4}

Where $X=h\nu/kT$, $X_{\text{max}}=h\nu_{\text{max}}/kT$, and $r =$ number of atoms per molecule. At low temperatures ($T<\theta_D/10$ K), it may be approximated by

$$C_V = 1944r \left( \frac{T}{\theta_D} \right)^3 \frac{1}{\text{mol K}} = \beta T^3$$  \hspace{1cm} \text{Eq. 5}
However at low temperatures the electronic contribution to specific heat becomes significant [44]. This term can be added to the Debye model to give the total specific heat at low temperatures.

\[ C = \beta T^3 + \gamma T \]  
\[ \text{Eq. 3} \]

For ferromagnetic and ferrimagnetic materials a term \( \delta T^{(3/2)} \) would also contribute [44]. For antiferromagnetic materials a term \( \xi T^3 \) would contribute, however it is difficult to differentiate this term from the phonon contribution. This approximation works well up to 80 K for most metals. [44]

The agreement between Debye theory and experimental results is remarkably good for many solids, specifically at \( T < \theta_D = h\nu_{\text{max}}/k \) (\( \theta_D \)-Debye characteristic temperature). [45] Debye temperatures typically are between 100 and 400 K, such as Niobium with a body center cubic structure has a \( \theta_D = 250 \) K. [43] Though some materials are can have much higher Debye temperatures such as diamond with a Debye temperature of 2000 K. [43] It also recovers the Dulong-Petit law at high temperatures [44]. However at intermediate temperatures if suffers from inaccuracies due to theoretical simplifications. Debye’s theory was enhanced by Born, Blackman and Karman [45] who considered inter-atomic forces and calculated the frequency spectrums of the lattice vibrations in more detail.

1.3 Guide to this database

Typically \( C_p \) is measured experimentally and \( C_V \) is derived theoretically, see reference [44] for full details. It is useful to note that the difference between \( C_p \) and \( C_V \) at low temperatures is quite small. It is usually \( \sim 1\% \) at temperatures near \( \theta_D/2 \). [43] So for most purposes, we can use the two interchangeably at cryogenic temperatures.

Data is shown as tables, equations, and graphs. Analytical equations have been provided when possible. The objective was to provide a reliable equation for all materials across the full cryogenic temperature range. Graphs and figures comparing different data sets are presented in the appendix.

Data is organized into insulators, superconductors, and metals chapters. Every data set is labeled with a number in brackets that corresponds to a reference. Reference can be found in the bibliography. Equations are labeled with temperature and magnetic ranges that they are applicable. When available, uncertainty is kept with the raw data, in tabulated form. If uncertainty analysis is unavailable it must be implied form significant digits. Certain materials have special parameters that greatly affect their properties. Details in these cases are given below.

This database is not exhaustive. It does give information on the validity of different datasets. Certain materials require particular attention to the specific circumstances, often time cause large variations between samples of the same materials. Other materials have simple behavior and there is good agreement between measurements.
2 Material properties

2.1 Insulation: G10

G10 insulation is commonly used in prototypes of Nb$_3$Sn accelerator magnets. A name of G10 is used for composition consisting of epoxy (resin) and fiberglass. G-10 is a loose definition of fiberglass and resin and does not describe a particular material; it is a NEMA specification describing electrical and mechanical properties [50]. The actual composition is not well defined. G-10’s thermodynamic properties will change for different manufactures. G-10’s properties should be a function of the fiberglass resin ratio. Studies by Imbasciati [30] have made attempts to estimate this ratio. G-10 CR is specifically designed for cryogenic applications. G-10CR’s thermodynamic properties are consistent [50]. The data in figures ? and ? confirm this.

2.1.1 Specific heat of G10

G10 data for the specific heat is summarized in figures ? and ?. Due to the limited sources of data on G10, data was also gathered on related materials of epoxies and glass. It is suspect that G10 should have a specific heat capacity between glass and the epoxy. Fermilab uses CTD-101K Epoxy and S-2 glass for the Nb$_3$Sn based magnets. The G10 is also painted with a ceramic binder (CTD 1008x) applied on the coils after winding.

Specific heat data could not be found on CTD-101K. Data on other epoxy was gathered but shown considerable variation between brands. From figure ?, it can be seen that glass has a lower specific heat than all G-10 data at all but one epoxy dataset.

The only study that could be found on the ceramic binder was done by Imbasciati [30]. However this study was done with ceramic fibers rather the glass fibers.


$$C = 10^{-2.4083 + 7.6006 \times 10^{-2.8982 \times x^2 + 7.3301 \times x^2 - 4.2386 \times x^4 + 1.4294 \times x^5 - 2.4396 \times x^6 + 0.015236 \times x^7} \frac{J}{kg \cdot K}$$

Where $x = \log_{10}(T)$, valid for $1 < T < 300$ K, equation has a 2% error from data and it is extrapolated below 4 K. Comparison to other data below 4 K shows good agreement, justifying the extrapolation.

2.1.2 Heat conductivity of G10

This is the largest uncertainty of all material properties. This can not be estimated from epoxy and glass material properties [30] just as in the cause of specific heat G-10 CR is consistent between datasets while G-10 is divergent. In addition a factor of the ceramic binder CTD-1002X affecting the G-10 thermal conductivity. There is no experimental data on CTD-1002X in G10. It effects remain unknown. Though G10
thermal conductivity remains below .3 W/(m*K) under 20 K for all data. According to [33] the conductivity is strongly depended on the treatment of the glass mica tape.

Experiments from [33] show that it can vary by a factor of 4 at 4 K from treatment processes alone.

G10’s thermal conductivity is directional in the cable, thus typically two values are given. The normal direction is of most interested for modeling of superconducting magnets.

Reference [22]

\[ k = 0.0179T - 0.0129 \]

From 0 K to 4 K the data was extrapolated. The values were computed by a computer program "MATPRO" the accuracy of these values is within an error of about 10%-20%.

Reference: G-10CR(Normal) tabulated data from [43] was fitted to the following equations

\[
k = -3.6229999163833060E-14x^6 + 2.996579202182310E-11x^5 - 9.687545615732300E-09x^4 + 1.599284085411630E-06x^3 - 1.452811767382280E-04x^2 + 8.279441504868440E-03x + 4.082144325604080E-02
\]

Valid for 4<T<150 K, \( R^2 = 9.999974932079730E-01 \)

Reference: G-10 CR Norm [13]

\[
k=10^{(-4.1236+13.788*Log(T)-26.068*Log(T)^2+26.272*Log(T)^3-14.663*Log(T)^4+4.4954*Log(T)^5-0.6905*Log^6+0.0397*Log(T)^7)}
\]

Valid for 10<T<300 K

2.2 Insulation – Polyimide

There are number of brands of Polyimide, LARP magnets use the brand kapton, polypyromellitimide (PPMI). [51] There are a number of different grades of kapton.

2.2.1 Specific heat of Polyimide

There is limited data on polyimide heat capacity. Kapton heat capacity data diverges at low temperatures. It could be due to different grades of kapton failing to be reported.

References: Kapton [13], [25], [51]
C =10^(-1.3684+.65892x+2.8719x^2+.42651x^3-3.0088x^4+1.9558x^5-.51998x^6+.051574x^7)

Where x = log_{10}(T)

Valid for 4<T<300 K, curve fit 3 % error relative to data

2.2.2 Heat conductivity of Polyimide

Kapton of different grades have different thermal conductivities. [28] Few sources reported there grade of kapton. Most likely the grade was HN regular kapton.

References: Tabulated data from [13][25][40] was fitted to the following equations

\[ k = -5.549425651993340E-08x^6 + 3.218098410811410E-06x^5 - 7.30847987426390E-05x^4 + 8.081548087134060E-04x^3 - 4.310834101394770E-03x^2 + 1.124627001832080E-02x - 1.460019416713240E-03 \]

Valid for 1.9<T<15K, R² = 9.998980502202240E-01

References [13][25][51]

\[ k=10^{(5.73101-39.5199(logT)+79.9313(logT)^2-83.8572(logT)^3+50.9157(logT)^4-17.9835(logT)^5+3.42413(logT)^6-.27133(logT)^7} \text{ (W/(m*K))} \]

Valid for 4<T<300k

Reference [28]

\[ k=5.24+/-0.32*10^{(-3)}T^{(1.02+/-0.02)} \text{ (W/(m*K))} \]

Valid in 5<T<300 K. This data is higher than all other datasets. However it is a different makeup then HN regular kapton. It matches kapton H films well.

2.3 Superconductors and alloys - NbTi

2.3.1 Specific heat of NbTi

Niobium Titanium specific heat data exhibits the characteristic second order phase transition peak, figure ?. This peak shifts to the left with increasing magnetic field. Specific heat data diverges at higher temperature. This is likely due to different alloy compositions. Few sources reported information on elemental %.
Reference [17] dataset 3

\[ C_{pNC} = 0.0023 \times T^3 + 0.145 \times T \text{ (J/(kg*K))} \]
Valid for 4.2<T<20 K

\[ C_{pSC} = (0.0023 + 3 \times 0.145 / (9.1^2)) \times T^3 + (0.145 \times B / 14) \times T \text{ (J/(kg*K))} \]
Valid for T<9.1 K, 0<H<7 T

Reference [25]

\[ C_{NC} = 0.1546667 \times T + 0.002706667 \times T^3 \text{ (J/(kg*K))} \]
Valid for Tc<T<20 K, assuming density (kg/m^3)=6000

\[ C_{NC} = 0.165714286 \times T + 0.0029 \times T^3 \text{ (J/(kg*K))} \]
Valid for Tc<T<20 K, assuming density (kg/m^3)=5600

\[ C_{NC} = 6.9 - 1.307683333 \times T + 0.092285 \times T^2 + 1.996666667 \times 10^{-3} \times T^3 - 3.633333333 \times 10^{-5} \times T^4 \text{ (J/(kg*K))} \]
Valid for 20<T<50 K density (kg/m^3)=6000

\[ C_{NC} = 7.392857143 - 1.401089286 \times T + 0.098876786 \times T^2 + 0.002139286 \times T^3 - 0.000038929 \times T^4 \]
Valid for 20<T<50 K density (kg/m^3)=5600

\[ C_{NC} = -255 + 13.837 \times T - 1.193833333 \times T^2 + 0.000496 \times T^3 - 8.03333 \times 10^{-7} \times T^4 \text{ (J/(kg*K))} \]
Valid for 50<T<175 K density (kg/m^3)=6000

\[ C_{NC} = -273.2142857 + 14.82535714 \times T - 1.27910714 \times T^2 + 0.00531429 \times T^3 - 8.607142857 \times 10^{-7} \times T^4 \text{ (J/(kg*K))} \]
Valid for $50 < T < 175$ K density (kg/m$^3$) = 5600

$$C_{NC} = 206.6666667 + 2.284333333 * T - 0.00861 * T^2 + 1.549333333 * 10^{-5} * T^3 - 1.048333333 * 10^{-8} * T^4$$

Valid for $175 < T < 500$ K density (kg/m$^3$) = 6000

$$C_{NC} = 221.4285714 + 2.4475 * T - 0.009225 * T^2 + 1.66 * 10^{-5} * T^3 - 1.123214286 * 10^{-8} * T^4$$

Valid for $1 < T < 15$ K density (kg/m$^3$) = 6000

$$C_{SC} = 0.008183333 * T^3 + 0.010666667 * T * B \text{ (J/(kg*K))}$$

Valid for $1 < T < 15$ K density (kg/m$^3$) = 5600

$$C_{SC} = 0.008768571 * T^3 + 0.014428571 * T * B \text{ (J/(kg*K))}$$

Valid for $1 < T < 15$ K density (kg/m$^3$) = 5600

Reference [27]

$$C_{NC} = 2.3 * 10^{-3} T^3 + 0.145 T$$

Valid for $T < 30$ K

$$C_{SC}(\theta) = \beta_1 \theta^3 + \gamma_1 \theta \text{ [J · Kg}^{-1} · K^{-1}]$$

$$\beta_1 = 2.3 * 10^{-3} + 0.435 / (Tc0)^2 \quad \gamma_1 = 0.145 B / Bc$$

$$B_e(\theta) = (B_{e0} + 5) \times \left[ 1 - \frac{\theta}{\theta_{e0}} \right]^{-5} \times \left[ 1 - \frac{\theta}{\theta_{e0}} \right]^{4.5}$$

$$B_s(\theta) = B_{e0} \left[ 1 - \left( \frac{\theta}{\theta_{e0}} \right) \right]^{1.7}$$

Reference [29]

$$C_{vSC} = (0.87(B/14) * T + 4.464 * 10^{-2} * T^3) * (100^3/1000) / 6000 \text{ (J/(kg*K))}$$

Valid for density (kg/m$^3$) = 6000
\[ CV_{SC} = (0.87(B/14)T + 4.46410^{-2}T^3)(100^3/1000)/5600 \text{ (J/(kg*K))} \]

Valid for density (kg/m^3) = 5600

\[ CV_{NC} = (0.87T + 1.3810^{-2}T^3)(100^3/1000)/6000 \text{ (J/(kg*K))} \]

Valid for density (kg/m^3) = 6000

\[ CV_{NC} = (0.87T + 1.3810^{-2}T^3)(100^3/1000)/5600 \text{ (J/(kg*K))} \]

Valid for density (kg/m^3) = 5600

2.3.2 Heat conductivity of NbTi

References: Tabulated data from [17][22][40] was fitted to the following equations

\[ k = -0.00089085306962360T^3 + 0.016706386304553200T^2 - 0.044789876496699500T + 0.068105653491378900 \]

Valid 1 < T < 10K \( R^2 = 0.996958 \)

Reference: tabulated data from [17] Dataset 2 was fitted to the following equations

\[ k = 2.707071295071070E-14T^6 - 1.089542905638570E-10T^5 + 7.26142646553600E-08T^4 - 1.768885700474560E-05T^3 + 1.52357906200000E-03T^2 + 1.965743220116850E-02T + 6.411246994511720E-04 \]

Valid for 10 < T < 100K \( R^2 = 0.9994 \)

Reference: tabulated data from [17] Dataset 2 was fitted to the following equations

\[ k = -3.611887887990550E-08T^3 + 8.451698778158840E-05T^2 - 1.476621990221600E-02T + 6.425342077064450 \]

Valid 100 < T < 300 k, \( R^2 = 0.9964 \)
Reference [27]

\[ K(\theta) = \frac{L_0 \theta}{\rho_{\text{normal}}} = 4.375 \times 10^{-2} \theta [\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}] \]

Valid for 2<T<200 K

2.4 Superconductors and alloys - Nb₃Sn

2.4.1 Specific heat of Nb₃Sn

Nb₃Sn specific heat exhibits the second order phase transition peak, figure ?. The peak occurs at the onset of the superconducting phase [44]. The peak is a strong function of magnetic field. Increasing the magnetic field shifts the superconducting peak to the left. Higher magnetic fields result in a reduction of strand enthalpy.

Nb₃Sn specific heat data shows considerable divergences for both the superconducting state and the normal conducting state, figure ?. For example, at 10 K super conducting data diverges by approximately 19 J/(kg*K) and the normal conducting data diverges by 4 J/(kg*K). Though most data converges by 2 K, figure ?. The primary source of this spread is likely compositional variation of the diffused tin. [47] Different fabrication schemes result in different percentages of tin in Niobium, see figure ?. [46][47][48] Using figure ?, ?, and the Kopp-Neumann law we expect that with a increased Tin content would result in a increased the heat capacity of the normal conducting strand below 100 K and lower the heat capacity above 100 K. It is important to note figure ?, the upper black line represents tin and the lower black line represents niobium. Normal conducting Nb3Sn should lie between these two black lines, however only [16], [39], [27], [25], [17] dataset one lie between these lines.

There is also compositional variation within the filaments. So one would expect the inner and outer layer to have different tin percentage depended properties. Compositional variation in the superconducting state results in a softening of the superconducting peak, figure ?.[48] This is because different alloying percentages have different critical temperatures. [48]

LARP Nb₃Sn cables are fabricated with the internal-tin route. Data from a similarly fabricated cable would be ideal. However, compiled sources seldom present this information with the data. This is because there are a multitude of factors effecting the specific heat measurement. For instance the strands of ITER contain fewer sub-elements then Fermi Lab LARP Nb₃Sn stands.

Fermilab has used references [27] and [39] for their data sets. Reference [17] suggests using the equations form data set two. Without specific compositional knowledge it is suggested that an upper and lower value should be taken from the data.
Reference [16]

\[ C_{NC} = \frac{(11.06T + 0.155T^3 - 0.000065T^5)}{99.35} \]

Valid for 9 < T < 18 K, H = 0 T

\[ C_{NC} = \frac{(11T + 0.00015568T^3)}{99.35} \]

Valid for T < 12K, H = 0 T

Reference [17] Dataset 2

\[ C_{pNC} = 0.001 T^3 + 0.1 T \] (J/(kg*K))

\[ C_{psc} = \left( \frac{(A + B) (T/Tc)^2}{8040} \right) \] J/(kg*K)

\[ A = \frac{1500 C^2}{(2D - 1)} \]

\[ C = -0.46306 - 0.06783 Tc \]

\[ D = \frac{27.2}{(1 + 0.34 Tc/Tc0)^2} \]

\[ B = 0.0075475 Tc^2 \]

Valid for 4.2 < T < 20 K, 0-7 T

Reference [17] Dataset 2

\[ C_{psc} = \frac{((A + B) (T/Tc)^3)}{8040} \]

\[ A = \frac{1500 C^2}{(2D - 1)} \]

\[ C = -0.46306 - 0.06783 Tc \]

\[ D = \frac{27.2}{(1 + 0.34 Tc/Tc0)^2} \]

\[ B = 0.0075475 Tc^2 \]

Valid for T < Tc = 7 K

\[ C_{p \text{Transition } SC} = \frac{(T - Tcs)(Tc - Tcs) \cdot C_{p \text{norm}}(T) + (1 - (T - Tcs)(Tc - Tcs)) \cdot C_{psc}}{(Tc - Tcs)} \]

For Tcs = 7K < T < Tc = 9K [17]

\[ C_{pNC} = 0.0075475 T^2 \]

\[ T, Tc < 10K \]

\[ C_{pNC} = (0.3 + 0.00375 T^2)/0.09937 \]

\[ 10 < T < 20K \]

Valid for Tcs < T < Tc

\[ C_{pNC} = 0.0075475 T^2 \]

\[ T, Tc < 10K \]
\[ C_{pNC} = \frac{(0.3 + 0.00375T^2)}{0.09937} \quad 10 < T < 20K \]

Valid for \( 20 \text{K} > T > T_c \)

Reference [27]

\[ C_{SC} = 4.2 \times 10^{-3}T^3 + 1.83T \; (J/(kg*K)) \]

Reference [25]

\[ C = 22.68T^3 + 988.2 \; (J/(m^3*K)) \]

2.4.2 Heat conductivity of Nb\(_3\)Sn

Reference [27]

\[ \rho_{Nb_3Sn}(\theta) = \min(6.0 \times \theta^2, 1.75 \times \theta^2 - 1.09 \times \theta + 3.11) \; [W \cdot m^{-1} \cdot K^{-1}] \]

Comparison with other data shows that this is inaccurate.

Reference: tabulated data from [22][17] dataset 3&2 was fitted to the following equations

\[ k = -1.906620820328610E-06x^5 + 4.842380418335070E-05x^4 - 2.014481796734910E-04x^3 + 6.56180188212804E-03x^2 - 1.660094463272800E-02x + 1.384398172179600E+00 \]

Valid for \( 2 < T < 20 \text{K} \), \( R^2 = 9.963323874710990E-01 \)

Reference: tabulated data from [17] dataset 3 was fitted to the following equations

\[ k = -2.365745696234404E-13x^6 + 2.41844482194210E-10x^5 - 9.782152807238660E-08x^4 + 1.976910315735690E-05x^3 - 2.039379799560000E-03x^2 + 8.96542341817190E-02x + 1.384398172179600E+00 \]

Valid for \( 20 < T < 300 \text{K} \), \( R^2 = 9.969291344533260E-01 \)
2.5 Metals - Stainless steel

2.5.1 Specific heat of Stainless Steel 304

References [13]

\[ C = 10^{(22.0061 - 127.5528x + 303.6470x^2 - 381.0098x^3 + 274.0328x^4 - 112.9212x^5 + 24.7593 \log x^6 - 2.239153x^7)} \]

Valid for 4<T<300 K

References [18][19]

\[ C = (465*T + .56*T^2)*10^{(-3)} \quad \text{(J/kg*K)} \]

Valid for .1<T<5 K

2.5.2 Heat conductivity of Stainless Steel 304

Reference [13]

\[ K_{10} = 1.4087 + 1.3982(\log_{10}T) + 0.2543(\log_{10}T)^2 - 0.6260(\log_{10}T)^3 + 0.2334(\log_{10}T)^4 + 0.4256(\log_{10}T)^5 - 0.4658(\log_{10}T)^6 + 0.1650(\log_{10}T)^7 + -0.0199(\log_{10}T)^8 \]

Valid for 1<T<300K, data range 4<T<300K.

2.5.3 Specific heat of Stainless Steel 316LN

Reference [17]

\[ C_p = .48 + 0.00075 T^3 \]

Valid for 1<T<10K

Tabulated data from [17] was fitted to the following equations

\[ C_p = -4.862348815137810E-06x^5 + 3.444074383409660E-04x^4 - 7.949135027672580E-03x^3 + 8.779089919403300E-02x^2 + 8.688409890100960E-02x + 6.933646148954720E-01 \]

Valid for 4<T<30 K, \( R^2 = 9.99999968471320E-01 \)
\[ C_p = -2.085598618982040E-13x^6 - 3.173418409805720E-09x^5 + 2.874635403118550E-06x^4 - 9.03402877521940E-04x^3 + 1.114639598744770E-01x^2 - 1.982593093780390E+00x + 1.167335212762210E+01 \]

Valid for 30<T<300 K, \( R^2 = 9.980263310755610E-01 \)

### 2.5.4 Heat conductivity of Stainless Steel 316LN

Reference: tabulated data from [17] dataset 1 & 2 was fitted to the following equations

\[ k = -7.59431235277220E-10x^6 + 9.276437079120190E-08x^5 - 4.107985169536380E-06x^4 + 5.916755214064800E-05x^3 + 1.346998877939300E-03x^2 + 6.932605599712440E-02x - 7.753724984925710E-02 \]

Valid for 2<T<50 K \( R^2 = 9.999749812317060E-01 \)

\[ y = 1.025127570486030E-16x^6 - 5.216067028341940E-14x^5 - 4.973995535243620E-10x^4 + 8.397255361034610E-07x^3 - 5.111722779212390E-04x^2 + 1.4146330909779250E-01x + 5.224030268254580E-01 \]

Valid for 50<T<300K, \( R^2 = 9.990762694676240E-01 \)

### 2.6 Titanium

#### 2.6.1 Specific heat of Ti

Specific Heat of Titanium

Reference [20]

\[ C_v=(71000+540*T^3)/10^6 \text{ (J/kg*K)} \]

Valid for 0<T<10K=\(T_{\text{max}}\). From figure ?, it can be see that this equation does not match well with data.

Reference: tabulated data from [19][20][22][24][43] was fitted to the following equations

\[ C_p = -4.187501509989030E-07x^6 + 1.871522770358070E-05x^5 - 2.772930183532200E-04x^4 + 2.384488858623970E-03x^3 - 5.970649812351070E-03x^2 + 7.977271559915520E-02x + 4.941024663526860E-03 \]
Valid for $1<T<20$ K, $R^2 = 9.999999193097490E-01$

\[ C_p = 3.639064543627110E-09x^6 - 6.199156261290500E-07x^5 + 1.822716742346570E-05x^4 + 1.351287013378180E-03x^3 - 2.315364264511310E-02x^2 + 2.257414222851980E-01x - 2.365529197364100E-01 \]

Valid for $20<T<70$ K, $R^2 = 9.999698136624870E-01$

\[ C_p = 1.345199296029590E-11x^6 - 1.5409641711100770E-08x^5 + 6.935071989007290E-06x^4 - 1.516633872522780E-03x^3 + 1.527963939856350E-01x - 2.750033746607780E+00x + 9.053558717787490E+00 \]

Valid for $70<T<300$ K, $R^2 = 9.997953432556180E-01$

### 2.6.2 Heat conductivity of Titanium

**Reference:** tabulated data from [24] was fitted to the following equations

\[ k = -4.146563318366350E-13x^6 + 4.076998258715820E-10x^5 - 1.570700596710640E-07x^4 + 3.056163499969890E-05x^3 - 3.239582681682280E-03x^2 + 2.007559945258790E-01x - 1.188200752254610E+00 \]


**Reference:** tabulated data from [43] was fitted to the following equations

\[ k = -4.151978065021750E-06x^3 + 3.976498237367890E-04x^2 + 4.076654915785600E-02x - 1.011750881316930E-01 \]

Valid $20<T<100$ K, Ti (6%Al-4%V), $R^2 = 1.000000000000000E+00$

\[ k = -6.634091860953030E-07x^3 + 3.785341337428670E-04x^2 - 4.712159709618850E-02x + 5.390227558285250E+00 \]

Valid for $100<T<300$ K, Ti (6%Al-4%V), $R^2 = 9.99999999989210E-01$
2.7 Niobium

2.7.1 Specific heat of Nb

Can be superconducting, but this data will most likely be unneeded in a magnetic due to the low critical field of niobium.

Reference [20]

\[ C_{\text{vNC}} = \frac{(85000 + 640 \times T^3)}{10^6} \text{(J/kg*K)} \]

Valid for 0 < T < 1 K

Reference [17]

\[ C_{\text{NC}} = 0.012 \times T^3 + 0.085 \times T \text{(J/(kg*K))} \]

Valid for 4 < T < 30 K

References: tabulated data from [24],[20],[19],[17],[15],[14] was fitted to the following equations

\[
C_{\text{NC}} = 1.854335962359220 \times 10^{-11} T^6 - 1.884320078112840 \times 10^{-5} T^5 + 7.377604897773620 \times 10^{-4} T^4 - 1.366177928330710 \times 10^{-3} T^3 + 1.112919864921200 \times 10^{-1} T^2 - 1.172816904749200 E + 00 T + 3.016294511908200 E + 00 \\
\text{Valid for 30 < T < 300 K, } R^2 = 9.988420465017880 E - 01
\]

\[
C_{\text{SC}} = -1.765844001244690 \times 10^{-9} T^6 + 5.070570594796210 \times 10^{-5} T^5 - 5.712932158076000 \times 10^{-4} T^4 + 3.481390764500250 \times 10^{-3} T^3 - 7.027787824699770 E - 02 T^2 + 4.631436617273720 E - 02 T - 3.082501121753010 E - 03 \\
\text{Valid for 0 < T < 9 K, } R^2 = 9.998435299042530 E - 01
\]

2.7.2 Heat conductivity of Nb

References: tabulated data from [22],[24],[17] dataset 3 was fitted to the following equations

\[ k = 50 \]

\[
k = 5.423769768026070 E - 09 T^6 + 2.181439693164500 E - 06 T^5 - 3.560634679318170 E - 04 T^4 + 3.002985569274300 E - 02 T^3 - 1.361012454973540 E + 00 T^2 + 3.011317741639440 E + 01 T - 1.639796816323040 E + 02 \\
\text{Valid for 20 < T < 100 K, } R^2 = 9.989740959466120 E - 01
\]
\[ y = -1.632652375249680 \times 10^{-05}x^6 + 9.958053412759680 \times 10^{-04}x^5 - 2.289708012806100 \times 10^{-02}x^4 + 2.437048464098100 \times 10^{-01}x^3 - 1.272840180504940 \times 10^0x^2 + 8.104067656280450 \times 10^0x + 3.269170199763490 \times 10^0 \]

Valid for \(2<T<20\) K, \(R^2 = 9.999997188282660 \times 10^{-01}\)

### 2.8 Tin

#### 2.8.1 Specific heat of Sn


\[ C_v = (14700 + 2210T^3) / 10^6 \text{ (J/kg*K)} \]

Valid for \(0<T<2\) K [20]

**Reference [24] White Tin: Tabulated data from [24] was fitted to the following equations.**

\[
C_p \text{ J/(kg*K)} = 8.651957877248150 \times 10^{-05}x^6 - 3.349530150529740 \times 10^{-03}x^5 + 4.686885801594090 \times 10^{-02}x^4 - 2.867543747755120 \times 10^{-01}x^3 + 8.542735671570510 \times 10^0x^2 - 1.137213002845870 \times 10^1x + 5.440992180789550 \times 10^{-01}
\]

Valid for \(1<T<10\) K, \(R^2=9.999989331180580 \times 10^{-01}\)

\[
C_p \text{ J/(kg*K)} = y = 1.133333333331880 \times 10^{-04}x^4 - 1.09999999999998150 \times 10^{-02}x^3 + 3.9716666666665410 \times 10^{-01}x^2 - 2.7249999998600 \times 10^0x + 5.50000000055960
\]

Valid for \(10<T<30\) K, \(R^2=9.999999999912710 \times 10^{-01}\)

\[
C_p = 10^{a + b \cdot x + c \cdot x^2 + d \cdot x^3 + e \cdot x^4 + f \cdot x^5 + g \cdot x^6 + h \cdot x^7 + i \cdot x^8 + j \cdot x^9 + k \cdot x^{10} + l \cdot x^{11}} \text{ J/kg*K}
\]

where \(x = \log_{10}(T)\). The coefficients for equation are given in table 1.

**TABLE 1**

<table>
<thead>
<tr>
<th></th>
<th>1.55718652083960E-05</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td><strong>b</strong></td>
</tr>
<tr>
<td>3.28428483506997E-05</td>
<td>7.06924168936027E-05</td>
</tr>
</tbody>
</table>
Valid for 25<T<300 K $R^2=0.999695774143388$

### 2.8.2 Heat conductivity of Tin

Tin heat conductivity increases at low temperatures, figure ?.


\[
k = -4.879359708933550E-02x + 1.903247692450810E+02
\]

Valid for 35<T<200K, Tin: 99.995% pure, single crystal, All data between 35<T<295 K is extrapolated, $R^2 = 5.48424541253090E-01$

\[
k = 180 \text{ W/(m*K)}
\]

Valid for 200<T<300K, extrapolated data.

\[
k = 1.139646700283330E-04x^6 - 1.543856378711080E-02x^5 + 8.544894414597910E-01x^4 - 2.485223028318840E+01x^3 + 4.043972973968770E+02x^2 - 3.550721992769150E+03x + 1.371471776541710E+04
\]

Valid for 10<T<30K, pure, single crystal, $R^2 = 1.000000000177030$

\[
k = -1.420168159820610E-01x^6 + 5.822864544461480E+00x^5 - 9.007839438028170E+01x^4 + 6.683951905751600E+02x^3 - 2.521412335171820E+03x^2 + 4.570981105430050E+03x - 5.394627478686740E+02
\]

Valid for 2<T<10 K, pure, single crystal, $R^2 = 9.999999492963420E-01$

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1.54816059121578E-04</td>
</tr>
<tr>
<td>e</td>
<td>6.07112089853614E-01</td>
</tr>
<tr>
<td>f</td>
<td>-4.24278700670351E-01</td>
</tr>
<tr>
<td>g</td>
<td>4.95713065603732E-02</td>
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<tr>
<td>h</td>
<td>5.85746556006985E-02</td>
</tr>
<tr>
<td>i</td>
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<tr>
<td>j</td>
<td>5.16442717073555E-03</td>
</tr>
<tr>
<td>k</td>
<td>-1.46819770508911E-03</td>
</tr>
<tr>
<td>l</td>
<td>-1.19012436777440E-06</td>
</tr>
</tbody>
</table>
2.9 Copper
Only sparse data was gathered on copper. Significant data can still be extracted from references.

2.9.1 Specific heat of Copper

Datasets of specific heat of copper converge. Reference 13 can be used with confidence.

Reference [13]

\[ C_p = 10^{(-1.91844-0.15973*x + 8.61013*x^2 -18.996*x^3 + 21.9661*x^4 -12.7328*x^5 + 3.54322*x^6 -0.3797*x^7)} \]

Where \( x = \log_{10}(T) \) Valid for 4<T<300 K

2.9.2 Heat conductivity of Copper

Great caution need to be taken when using data on the thermal conductivity of copper. Copper conductivity is a strong function of RRR effects and magnetic fields. Reference [13] website and [17] contain detailed data.

2.10 Bronze

Bronze is an alloy of copper and tin. Copper content usually varies by 30 to 55 
%
.[33].

2.10.1 Specific heat of bronze

Specific Heat of Bronze

Specific heat of bronze is expected to lie between copper and tin.

Reference [17] Dataset2

\[ C_v =\frac{1}{M}(\gamma*T+D(x)) \]
\[ M(Sn)=1.187*Sn+.6354*(100-Sn) \]
\[ \gamma=.698+.00911*Sn-.00128*Sn^2+6.54*10^{-5}*Sn^3 \]
\[ \theta(Sn)=344-3.11*Sn \]
\[ x=\theta(Sn)/T \]

Where Sn is % tin content.
TABLE 2 (table description)

Where debye function \( D(x) \) is given numerically

<table>
<thead>
<tr>
<th>( x )</th>
<th>( D(x) ) (J/kmol/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24915.72</td>
</tr>
<tr>
<td>1</td>
<td>23723.28</td>
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<tr>
<td>2</td>
<td>20585.28</td>
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<td>3</td>
<td>16526.8</td>
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<td>4</td>
<td>12535.26</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>6619.088</td>
</tr>
<tr>
<td>7</td>
<td>4757.208</td>
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<tr>
<td>8</td>
<td>3443.432</td>
</tr>
<tr>
<td>9</td>
<td>2527.136</td>
</tr>
<tr>
<td>10</td>
<td>1891.168</td>
</tr>
<tr>
<td>11</td>
<td>1443.48</td>
</tr>
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</tr>
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<td>13</td>
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<td>14</td>
<td>707.096</td>
</tr>
<tr>
<td>15</td>
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<tr>
<td>16</td>
<td>472.792</td>
</tr>
<tr>
<td>17</td>
<td>395.388</td>
</tr>
<tr>
<td>18</td>
<td>333.046</td>
</tr>
<tr>
<td>19</td>
<td>283.257</td>
</tr>
<tr>
<td>20</td>
<td>243.09</td>
</tr>
<tr>
<td>22</td>
<td>182.422</td>
</tr>
<tr>
<td>26</td>
<td>110.458</td>
</tr>
<tr>
<td>30</td>
<td>71.965</td>
</tr>
</tbody>
</table>

Valid for \( 0<T<300 \text{ K} \), by comparison to other data figure ?, this equation works well between \( 0<\text{Sn}<60\% \).

2.10.2 Heat conductivity

\[
y = 10 \left( .4145 + 1.563 \log_{10} T - .2285 (\log_{10} T)^2 - .3234 \times \text{Sn} + .02500 \times \text{Sn}^2 \right) \text{ (W/(K*m))}
\]

Where Sn is in wt%, valid for \( 1<T<300 \text{ k} \), No significant effect of annealing, \( \text{Sn}<7.3 \text{ wt\%}[17] \)
3 Bibliography

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'[39] Eric Marscin,Ryuji Yamada "3D ANSYS Quench Simulation Program with Full Magnet Coil Geometry" Fermi National Accelerator Laboratory TD-03-043 09/08/03


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http://www.matweb.com/search/DataSheet.aspx?MatGUID=ab35b368ab9c40848f545c35bd1a672

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"Note that not all data has been extracted."

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BETWEEN 4.2 AND 300K, WITH AND WITHOUT ALUMINA PARTICLES AS FILLER" August 1990
4 Appendix

4.1 Appendix 1

Kapton Specific Heat Capacity

Temperature (K): 0 2 4 6 8 10 12 14 16

C (J/(kg·K)): 0 5 10 15 20 25 30 35

Kapton Specific Heat Capacity

Temperature (K): 0 50 100 150 200 250 300 350

C (J/(kg·K)): 0 100 200 300 400 500 600 700 800
G10 Specific Heat Capacity

[43] G-10CR
[13] G-10 CR (Fiberglass Epoxy) Equation:
[22] G10
[39] G-10
[19] Glass Fiber Reinforced Polyester
[43] Glass/resin (s901Glass/NASA resin 2)
[24] SPECIFIC HEAT of VITREOUS SILICA(Silica Class, Quartz Class.)
[43] Epoxy (stycast 2850FT)
[43] Epoxy (CY221)
G10 Specific Heat Capacity

- [13] G-10 CR (Fiberglass Epoxy) Equation:
- [22] G10
- [39] G-10
- [24] SPECIFIC HEAT of VITREOUS SILICA (Silica Class, Quartz Class.)
- [43] Glass/resin (s 901 Glass/NASA resin 2)
- [40] Epoxy
- [43] Epoxy (stycast 2850FT)
- [43] Epoxy (CY221)

Temperature (K)

Titanium Specific Heat Capacity

For 1 < t < 300 K

- [20][24]
- [22]
Titanium Specific Heat Capacity

For 25<T<293 K [19]

Bronze Specific Heat Capacity

- Bronze [17],[22]
- Bronze [17] Equation:
- Bronze and Copper [39]
- Copper [39]
- White Tin [24]
Bronze Specific Heat Capacity

C (J/(kg·K))

Temperature (K)

Bronze [17],[22]
Bronze [17] Equation:
White Tin [24]

Copper Specific Heat Capacity

C (J/(kg*K))

Temperature (K)

Copper specific heat capacity [13] C (J/(kg*K))
[39]
[43]
Copper Specific Heat Capacity

Niobium Specific Heat Capacity
Niobium Specific Heat Capacity

- Niobium Normal [14][15][17][19][20][24]
- Niobium Super [14][15][17][19][20][24]
- Niobium Normal [17] Equation:
- Niobium Normal [43]

Tin Specific Heat Capacity

- White tin [20]
- White Tin [22]
- White Tin [24]
304 stainless steel specific heat capacity

- Equation 1: [13]
- Equation 2: [18][19]
- [19]
- [43]

Temperature (K)

C (J/(kg·K))
316LN stainless steel specific heat capacity

C (J/(kg·K)) vs. Temperature (K)

- [17] dataset 1 & 2
- [17] Equation:
- [17] Dataset 2
Nb3Sn specific heat capacity

- Nb3Sn specific heat capacity
  - [16],[12],[17],[25],[22],[27] Cpnormal J/(kg*K)
  - Cpsuper J/(kg*K)

- Nb3Sn Normal H = 0 T [16] Equation:

- Nb3Sn Normal H = 0 T [16] Equation:

- Nb3Sn [17], Normal Equation:

- [17], Super B = 11T equation

- [17] super B = 2 T equation

- Nb3Sn [17] T<Tcs Equation CSuper (J/(kg*K))

- Nb3Sn [17] Tcs<T<Tc Equation CpTransitionSuper

- Nb3Sn [17] T>Tc Equation Cnormal

- Nb3Sn Super [27] Equation:

- Nb3Sn [39]
Nb3Sn specific heat capacity

Temperature (K)

C (J/(kg·K))

- Nb3Sn [17], Normal Equation:
- Nb3Sn [39]
- Nb3Sn Normal [27]
- Nb3Sn Mixed Super/Normal [25],[17]
- Nb3Sn Normal H=0 [12]
- Nb3Sn Normal H=0 T [16]
- White Tin [24]
- Niobium [24]
Nb3Sn specific heat capacity

Equation:

\[ C_{\text{Nb3Sn}}^{\text{Normal}}(T) = \begin{cases} \frac{1}{3}T^3 & T < T_{cs} \\ \frac{1}{2}T^2 & T_{cs} < T < T_c \\ \frac{1}{2}T^2 & T > T_c \end{cases} \]

\[ C_{\text{Nb3Sn}}^{\text{Super}}(T) = \begin{cases} \frac{1}{3}T^3 & T < T_{cs} \\ \frac{1}{2}T^2 & T_{cs} < T < T_c \end{cases} \]

\[ C_{\text{Niobium}}(T) = \begin{cases} \frac{1}{3}T^3 & T < T_{cs} \\ \frac{1}{2}T^2 & T_{cs} < T < T_c \end{cases} \]
Figure 1: [23] Nb3Sn magnetic field data, a 2nd order phase transition to superconducting shifts with changing magnetic field.

Figure 2: [23] softening of the 2nd order peak due to imperfect composition.
Superconducting and normal NbTi in 0 - 7 T

NbTI Specific Heat Capacity

Superconducting and normal NbTi in 0 - 7 T

[C (J/(kg·K))]

T (K)

NbTi [17],[19],[22]
NbTi [25] 6000 kg/m^3
NbTi [25] 5600 kg/m^3

[C (J/(kg·K))]

T (K)
G-10 CR (Fiberglass Epoxy) specific heat capacity
Polyimide (Kapton) specific heat capacity
Niobium Cnormal
Titanium specific heat capacity
Bronze Sn=10%
Tin specific heat capacity
304 stainless steel specific heat capacity
316LN stainless steel specific heat capacity
NbTi specific heat capacity
Nb3Sn specific heat capacity
Copper specific heat capacity
\[ y = -4 \times 10^{-14} x^6 + 3 \times 10^{-11} x^5 - 1 \times 10^{-8} x^4 + 2 \times 10^{-6} x^3 - 0.0001 x^2 + 0.0083 x + 0.0408 \]

\[ R^2 = 1 \]

\[ y = 0.0179 x - 0.0129 \]

\[ R^2 = 1 \]
Nb3Sn heat conductivity

\[ \lambda \text{ (W/(K*m))} \]

\[ T \text{ (K)} \]

[17] Dataset 3
[17] Dataset 2
[22]
**NbTi specific heat conductivity**

\[ y = -0.000195534180482251x^3 + 0.005970296759573430x^2 - 0.002875000875733270x + 0.028412220532686200 \]

\[ R^2 = 0.999028461721906000 \]
304 stainless steel heat conductivity

Hidden in the heat capacity section of [24]

[22]
**316LN heat conductivity**

\[ y = 1.025127570486030E^{-16} - 5.216067028341940E^{-14}x^6 + 8.397255361034610E^{-07}x^3 - 5.11722779212390E^{-04}x^2 + 1.414633090779250E^{-01}x - 5.224030268254580E^{-01} \]

\[ R^2 = 9.990762694676240E^{-01} \]

4<T<1144K [17] Data set 1
2<T<282K [17] Data set 2
[17] Dataset 1 & 2

**316LN heat conductivity**

\[ y = -7.594312235277220E^{-10}x^6 + 9.276437079120190E^{-08}x^5 - 4.107985169536380E^{-06}x^4 + 5.916755214064800E^{-05}x^3 + 1.346998877939300E^{-03}x^2 + 6.932605599712440E^{-02}x - 7.753724984925710E^{-02} \]

\[ R^2 = 9.999749812317060E^{-01} \]
Tin heat conductivity

$k (W/(m*K))$ vs $T (K)$

- Blue diamonds: [24]
- Red squares: [22]
Titanium heat conductivity

20<T<250 K extrapolated to 15K and 300 k [24]
Heat Conductivity

- Dariusz: Nb3Sn
- Giorgio: NB3SN
- Polyimide
### 4.2 Appendix 2

#### Table 3: Density of materials

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<th>G10</th>
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<th>Polyimide</th>
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<td>Ref</td>
<td>ρ (kg/m³)</td>
<td>Ref</td>
<td>ρ (kg/m³)</td>
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<th>NbTi</th>
<th>Nb3Sn</th>
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<td>ρ (kg/m³)</td>
<td>Ref</td>
<td>ρ (kg/m³)</td>
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